PHOTOMETRY

I. Introduction

Perception of visual stimuli is determined by many factors among which the dominant are the physical characteristics of the stimulus:

- the strength of stimulus, e.g. determined in terms of energy carried by a beam of light which strikes the eye and stimulates the retina
  and
- the wavelength of light which is related to specific sensitivity of the eye, dependent on the light wavelength.

Receptors of the retina, cones and rods, respond to only a narrow band of electromagnetic radiation. Substances accumulated in them (opsins – a protein family found in photoreceptor cells of the retina) absorb radiation only from the range 380 nm to 780 nm, called the visual radiation. That is why our eyes are sensitive neither to ultraviolet radiation (wavelength shorter than 380 nm) nor to infrared radiation (wavelength longer than 780 nm). Moreover, the activity of photoreceptors vary with the amount of light reaching the retina. Vision at low ambient light levels (e.g. at night – when the illumination conditions are unsatisfactory) is mediated by rods and is called the ***scotopic vision***. Rods have a much higher sensitivity to light than cones, however, the sense of color is essentially lost in the scotopic vision regime. In other words, at low light levels such as in a moonless night, objects lose their colors and only appear to have different levels of grey. On the other hand, as the illumination conditions increase, the scotopic vision gradually goes through the transitional phase called the ***mesopic vision*** (with impaired color sensation) to reach finally the ***photopic vision*** (e.g. during daylight conditions) when the vision is mediated by cones with full sensation of colors.
The above described properties of rods and cones are illustrated by the *luminous efficiency function* V(λ) or eye sensitivity function, shown in Fig.1, which illustrates how the sensitivity of the eye depends on the wavelength of light. In the photopic vision the peak sensitivity of the eye is in the green-yellow, at 555 nm (1 nm = 10^{-9} m); in the scotopic vision it shifts to 507 nm (a bluish-green color seen in photopic conditions). Values of the V(λ) are given in the appendix at the end of the text. They are needed for the conversion between radiometric and photometric units.

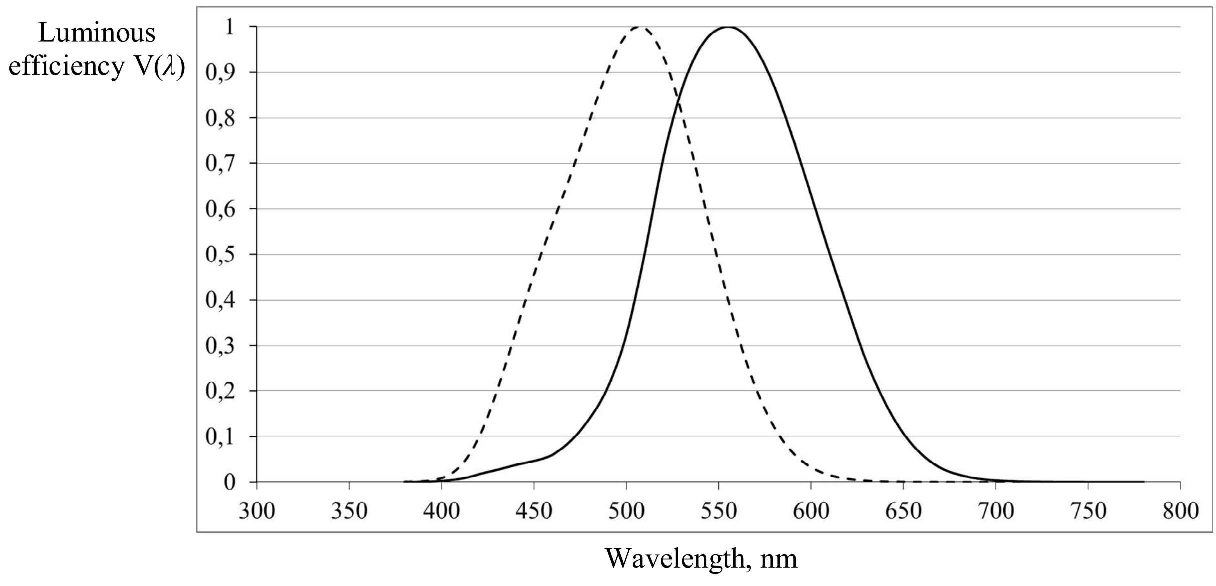


Fig. 1 Luminous efficiency function V(λ) shows how the sensitivity of the eye depends on the wavelength of light. Solid line – the photopic (cone-mediated daylight vision); broken line – the scotopic (rod-mediated night vision).

2. PHOTOMETRIC QUANTITIES AND THEIR UNITS

The physical properties of electromagnetic radiation are characterized by radiometric units. Using radiometric units we can characterize light in terms of physical quantities; for example, the number of photons, photon energy, and the radiant energy carried by light per unit time i.e. the power. However, when discussing the *light perception* by a human being the radiometric units are irrelevant. For example, infrared radiation ($\lambda > 780$ nm) causes no luminous sensation in the eye. The same concerns the ultraviolet radiation (10 through 380 nm).

To characterize the light and color sensation by the human eye, different types of units are needed. These units are called *photometric units*. Most of them have the adjective *luminous*. In the following text four of them will be defined, their units will be given and explained.

2.1. Luminous flux (luminous power)

The **luminous flux** $\boldsymbol{\Phi}$, represents the light power i.e. the amount of energy in unit time of a source *as perceived by the human eye*. The unit of luminous flux is the **lumen** (lm). It is defined as follows:

a monochromatic light source emitting an optical power of (1/683) watt at 555 nm has a luminous flux of 1 lumen (lm).

It is expressed by the following formula

$$\Phi = 683\frac{\text{lm}}{\text{W}}\int_{380\text{nm}}^{780\text{nm}} \Phi_{\text{E}}(\lambda)\cdot V(\lambda)\,d\lambda \qquad (1)$$

where: $\Phi_{\text{E}}(\lambda)=\dfrac{dP(\lambda)}{d\lambda}$ – power emitted at the wavelength λ in a unitary range of wavelength,

$V(\lambda)$ – luminous efficiency at the wavelength λ.

For a monochromatic light source it simplifies to:

$$\Phi = 683\frac{\text{lm}}{\text{W}}\times P(\lambda)\times V(\lambda)$$

Example

What is the luminous flux of a monochromatic source of light of 1 W power emitting radiation of wavelength:

a) 555 nm (green-yellow light)?

b) 490 nm (blue light)?

Solution:

a) the luminous efficiency $V(\lambda)$ of the eye at 555 nm takes maximum end equals 1 (see APPENDIX), thus the luminous flux is: $\Phi = 683\frac{\text{lm}}{\text{W}}\times 1\,\text{W}\times 1 = 683\,\text{lm}$.

b) the luminous efficiency $V(\lambda)$ of the eye at 490 nm equals 0.208, thus the luminous flux is:

$\Phi = 683\frac{\text{lm}}{\text{W}}\times 1\,\text{W}\times 0.208 = 142\,\text{lm}$

So, we can see that even though the amount of power reaching the eye is the same (1 W) the sensation of brightness of blue light (490 nm) is much lower than that of yellow green light (555 nm).

For the light sources emitting light at many wavelength, the determination of the luminous flux is more complex and needs detailed information about the power emitted by a source at every particular wavelength.

2.2 Luminous intensity

This photometric quantity characterizes a source of light and represents the intensity of a light source as perceived by the human eye. It represents the "amount" of light, i.e. the luminous flux, sent by a source into the unit *solid angle**. The unit of luminous intensity is **candela** (cd), which is the base unit of the International System of Units (SI unit). The definition of candela is as follows:

1 candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of 555 nm wavelength (or frequency 540×10^{12} hertz) emitting an optical power of (1/683) watt into the solid angle of 1 steradian (sr).

According to the above definition, one may find that the luminous intensity and the luminous flux are related quantities. The relation is represented by the equation (2):

$$I = \frac{\Phi}{\Omega} \tag{2}$$

where Φ - luminous flux in lumens,
Ω - solid angle in steradians.

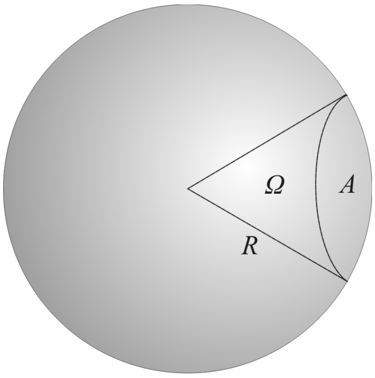


Fig. 2. A portion A of a sphere of radius R subtends a solid angle Ω.

***Solid angle:** *It is the angle subtended at the center of a sphere of a given radius R by an area A on the sphere's surface). The non-dimensional unit of the solid angle is steradian.*

The number of steradians in a given solid angle can be determined by dividing the area A on the surface of a sphere lying within the intersection of that solid angle with the surface of the sphere (when the apex of the solid angle is located at the center of the sphere) by the square of the radius R of the sphere:

$$\Omega = \frac{A}{R^2} \tag{3}$$

In general, any portion of a sphere surface with area A equal to squared radius (R^2) subtends one steradian. Or in other words: a s**teradian** "cuts out" an area of a sphere equal to squared radius of that sphere. If the surface A covers the whole sphere then the number of steradians is 4π:

$$\Omega = \frac{4\pi R^2}{R^2} = 4\pi$$

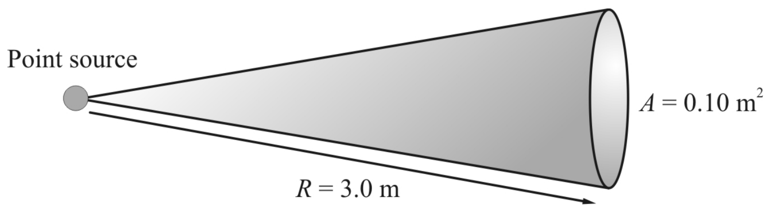
We can now return to the definition of the lumen and give it in an alternative way:

One lumen is defined as the luminous flux of light produced by a light source that emits one candela of luminous intensity over a solid angle of one steradian:

$$\Phi = I \cdot \Omega \qquad 1\,\text{lm} = 1\,\text{cd} \times 1\,\text{sr} \tag{4}$$

Example

What is the luminous flux within the solid angle shown on the drawing below if the point source luminous intensity is of 5000 cd?



Solution:

following (4): $\Phi = I \cdot \Omega = 5000\,\text{cd} \cdot \Omega$

$$\Omega = \frac{A}{R^2} = \frac{0.10\,\text{m}^2}{9.0\,\text{m}^2} = 0.011\,\text{sr}$$

Thus, the luminous flux equals: $\Phi = I \cdot \Omega = 5000\,\text{cd} \cdot 0.011\,\text{sr} = 55\,\text{lm}$

Example

(Laser pointer) The total power emitted by a laser pointer is 0.5 W. What is the luminous intensity of the pointer emitting light at 650 m if the spot of light at 10 meters distance has a 1.0-cm diameter.

Solution:

$$\Phi = 683\frac{\text{lm}}{\text{W}}\times 0.50\ \text{W}\times 0.107 = 36\ \text{lm}\ ,$$

$$\Omega = \frac{7.9\times 10^{-5}\ \text{m}^2}{100\,\text{m}^2} = 7.9\times 10^{-7}\ \text{sr}\ ,$$

$$I = \frac{36\ \text{lm}}{7.9\times 10^{-7}\ \text{sr}} = 4.6\times 10^{7}\ \text{cd}\ .$$

Example

What is the luminous flux emitted by a light source of 90 cd-luminous intensity into a hemisphere?

Solution:

$\Phi = I\cdot\Omega = 90\,\text{cd}\cdot 2\pi\,\text{sr} = 565\,\text{lm}$

3. Illuminance and the inverse square law

The **illuminance *E*** concerns light incident on a surface to illuminate it.
By definition:

illuminance is the luminous flux Φ incident per unit area A (Fig. 3). The illuminance is measured in lux. The lux is an SI unit used when characterizing illumination conditions of a surface:

$$E = \frac{\Phi}{A} \qquad \text{lux} = \frac{\text{lumen}}{\text{m}^2} \tag{5}$$

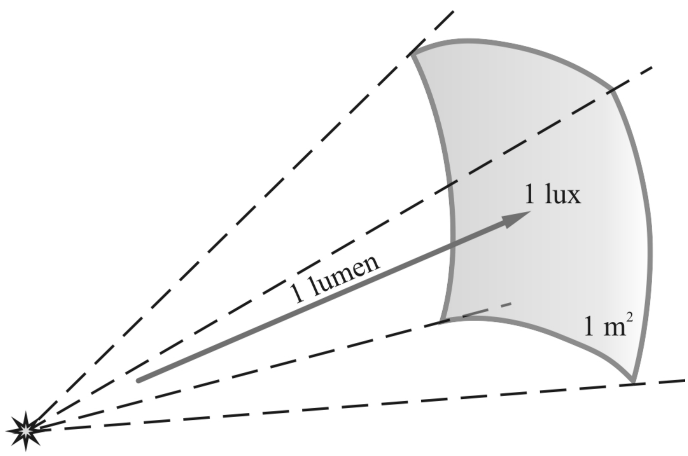


Fig. 3. If a luminous flux of 1 lumen is incident on a surface area of 1 m^2 the illuminance equals 1 lux.

First, let us consider light coming from a point source, emitted uniformly into all space. Since a sphere has the surface area of $4\pi R^2$, a source of luminous intensity I produces on the sphere surface the total illuminance of:

$$E = \frac{\Phi}{A} = \frac{I\cdot\Omega}{A} = \frac{I\cdot 4\pi}{4\pi R^2} = \frac{I}{R^2}$$

$$\boldsymbol{E = \frac{I}{R^2}} \tag{6}$$

The above expression is known as the inverse square law.

It is worth noticing that the inverse square law has a quite general meaning and does not concern the light only. In general we may say that any point source which spreads its influence equally in all directions without a limit to its range will obey the inverse square law. This conclusion comes from strictly geometrical considerations:

> *The intensity of the influence at any given radius R (or distance from the point source) is the source strength divided by the area of the sphere:*

$$\text{intensity of the influence} = \frac{\text{influence}}{R^2}$$

The inverse square law applies to diverse phenomena as for instance gravity (a point sources of gravitational force), electric field (a point source of electric field), acoustics (a point sound source) and many others.

Coming back to the issue of surface illuminance one may say, following the inverse square law, that the illuminance of a surface at a distance *R* is directly proportional to the luminous intensity of the source and inversely proportional to the square of the distance *R* from the source (Fig. 4).

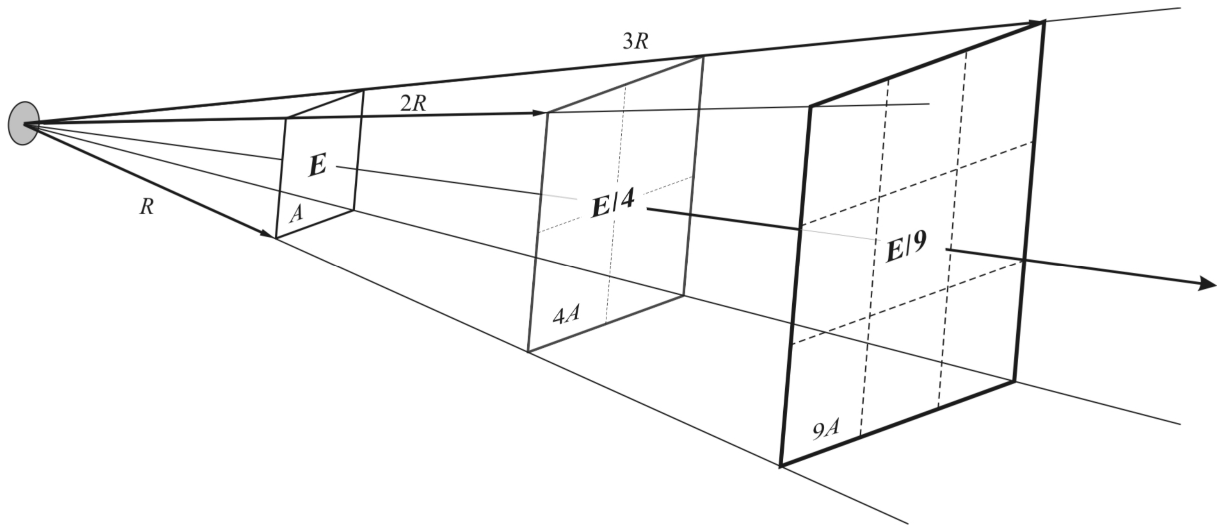


Fig. 4 The flux leaving a point source within any solid angle is distributed over increasingly larger areas, producing illuminance that decreases inversely with the square of the distance (eq. 6).

If the light source dimensions can no longer be approximated as a point source (for instance, a flat square ceiling mounted luminaires) the exponent "2" at *R* is no longer valid. When we proceed from a point source to an infinitely extended source, the inverse square law becomes:

$$E = \frac{I}{R} \tag{7}$$

which is now the inverse proportionality relationship. Thus, for realistic light sources the exponent in equations 6 or 7 is somewhere between 2 and 1.

So far we have assumed that the light reaches a surface perpendicularly. But if the angle of incidence α is different than 0 degrees the illuminance of the surface is less by the $\cos\alpha$ factor (Fig. 5):

$$E = \frac{I}{R^2} \cdot \cos\alpha \tag{8}$$

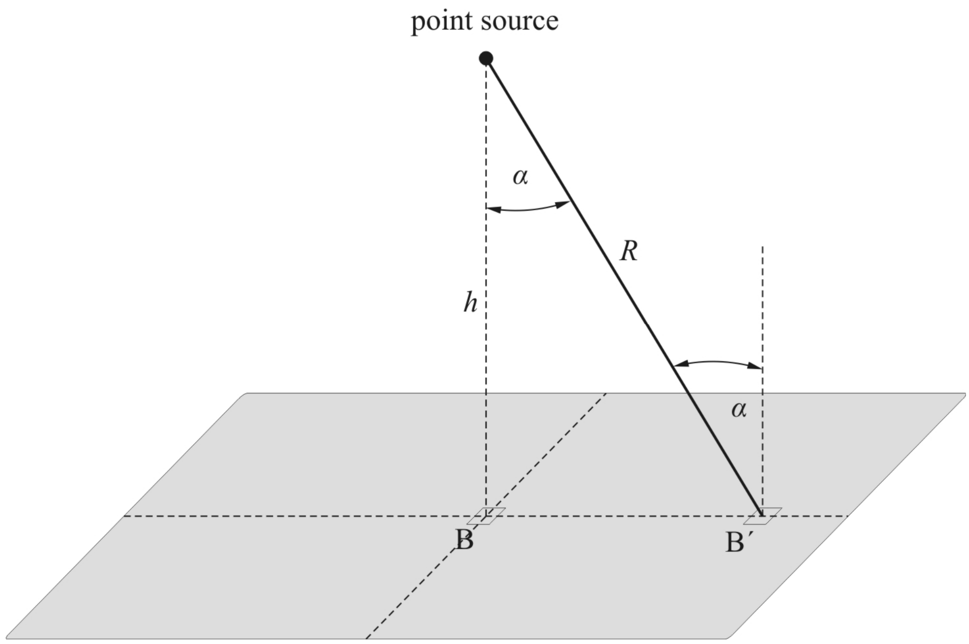


Fig. 5 The illumination at point B' is less than that in point B by the cosα factor.

Example

A 500-cd lamp (small enough to be a point source) is suspended 1.00 m above the center of a table. What is the illuminance:

a) in the center of the table (point B in Fig. 5)?

b) 0.80 m to the left of the center (point B' in Fig. 5)?

Solution:

a) from equation (8):

$$E_{\mathrm{B}} = \frac{500\,\mathrm{cd}}{(1.00\,\mathrm{m})^2}\cos 0° = 500\,\mathrm{lx}$$

b) first of all, applying the Pythagorean theorem, we find the distance R between the source and point B': $R = \sqrt{h^2 + (BB')^2} = 1.28\,\mathrm{m}$; next we calculate cosα = h/R so again from equitation (8) we have:

$$E_{\mathrm{B'}} = \frac{I}{R^2}\cdot\frac{h}{R} = \frac{500\,\mathrm{cd}}{(1.28\,\mathrm{m})^2}\frac{1.00\,\mathrm{m}}{1.28\,\mathrm{m}} = 238\,\mathrm{lx}$$

As we notice the illuminance is really less at point B' than that at point B.

Summing up: the illuminance is a major factor determining how reliably and easily our eyes are able to perform visual tasks, e.g. reading or working. Illumination conditions are standardized (see table 1).

Table 1. Standard recommendations related to dentistry*

Type of activity	illuminance in lux
General lighting - performance of visual tasks of high contrast or large size	500
At the patient	1000
Instrument tray, examination, performance of visual tasks of medium contrast and small size	500 - 1000
Oral cavity – performance of very prolonged and exacting visual tasks	5000 - 10000
white teeth matching	5000

*After the European Committee for Standardization (the EN 12464-1 standard): EN12464-1 schedule of illuminance and recommendations related to hospitals and healthcare buildings.

2.4. Luminance

The term *luminance* is very often confused with the term *illuminance*. Thus, before the formal definition of the luminance and its unit will be given let us try to understand this difference. A beam of light that is travelling towards a surface *illuminates* it. So the illuminance is measured as the amount of light striking a surface – the incident light. On the other side we have the luminance – which is the amount of light that leaves the surface in a given direction. Luminance is often used to characterize the emission from a diffuse surface or extended source. It indicates how much luminous power will be perceived by eye when viewing the surface from a particular angle.

Formal definition of the luminance is as follows:

> ***Luminance*** *(symbol L) is the luminous intensity per a unit area of a light emitting source (usually reflecting and/or diffusing surface) travelling in a given direction:*

$$L = \frac{I}{A \cdot \cos\alpha} \tag{9}$$

Or, substituting $\frac{\Phi}{\Omega}$ for I (see eq. 2):

$$L = \frac{\Phi}{A \cdot \cos\alpha \cdot \Omega}$$

we say that the luminance describes the amount of light (luminous flux Φ) that is emitted from a particular area, and falls within a given solid angle Ω. An SI unit for the luminance is candela per square meter:

$$[L] = \frac{\text{cd}}{\text{m}^2}$$

Luminance is thus an indicator of how bright the surface (or, usually extended source of light) will appear. Luminance is used for instance in the video industry to characterize the brightness of displays. A typical LCD displays have luminance between 200 and 300 cd/m^2.

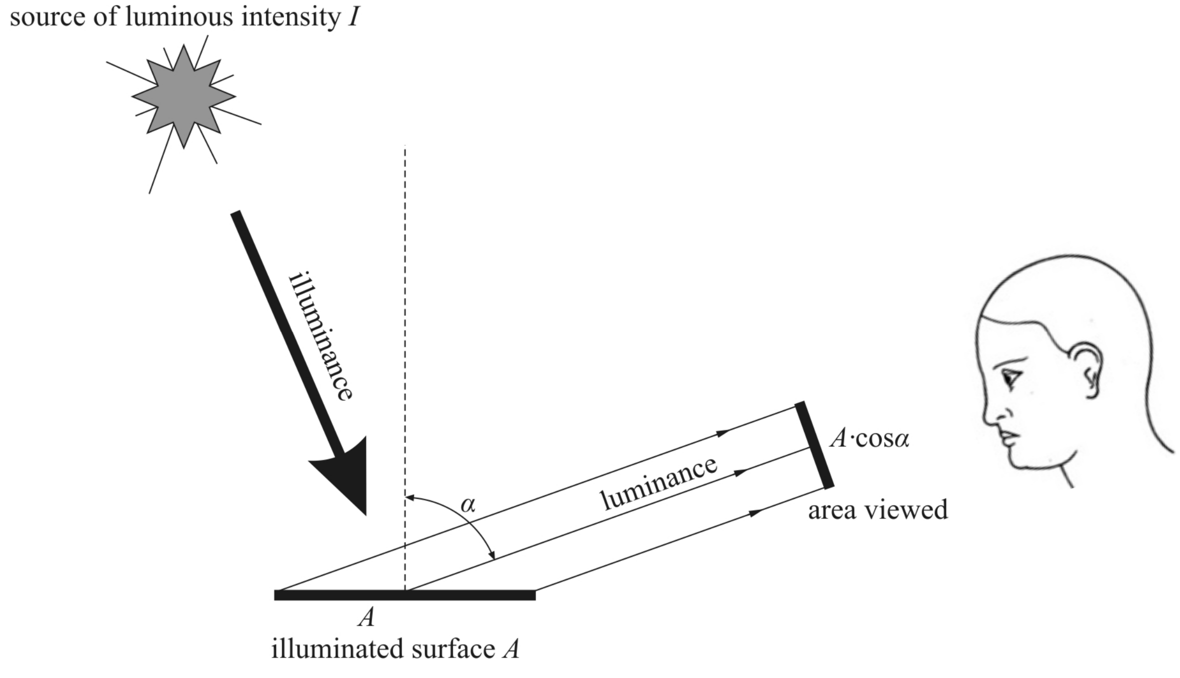


Fig.6. The luminance is what reaches your eyes from the illuminated surface.

Luminance remains the same regardless of the distance from the light source!

Summary of photometric quantities and their units is given in table 2.

Table. 2. SI photometry quantities.

Quantity		Unit		Notes
Name	Symbol	Name	Symbol	
Luminous flux	Φ	lumen (= cd·sr)	lm	$\Phi = 683 \frac{\text{lm}}{\text{W}} \int_{380\,\text{nm}}^{780\,\text{nm}} P(\lambda) \times V(\lambda) \mathrm{d}\lambda$ also called *luminous power*
Luminous intensity	I	candela (= lm/sr)	cd	$I = \frac{\Phi}{\Omega}$ an SI base unit
Illuminance	E	lux (= lm/m^2)	lx	$E = \frac{\Phi}{A}$ used for light incident on a surface
Luminance	L	candela per square meter	cd/m^2	$L = \frac{\Phi}{A \cdot \cos\alpha \cdot \Omega}$ Luminance remains the same regardless of the distance from the light source.

3. Comparative photometry

The branch of science dealing with measurements of the luminous intensity of light sources is called photometry. Systems for measuring light are called photometers. A schematic diagram of laboratory setup for comparative photometry is shown in Fig. 7.

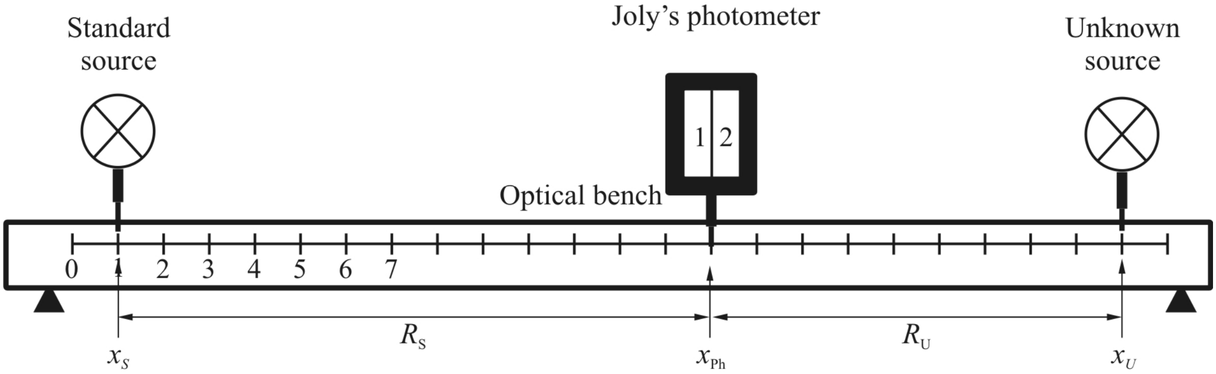


Fig. 7. A scheme of laboratory setup for comparative measurements of luminous intensity.

The comparative Jolly photometer consists of two equal paraffin wax blocks separated by a thin opaque sheet. The photometer is placed between two sources of light: the first is a standard or reference source, the second is an unknown source. During the measurement the photometer's position is adjusted until two wax blocks (1 and 2 in figure 7) appear equally bright. When the match of two sources is achieved, the illuminance E_S of the first block, due to the standard source equals the illuminance E_U of the second block due to an unknown source:

$$E_S = E_U$$

Now, following equation (6):

$$\frac{I_U}{R_U^2} = \frac{I_S}{R_S^2}$$

Knowing I_S R_S and R_U one can solve the above equation to determine the luminous intensity I_U of the unknown source:

$$I_U = I_S \frac{R_U^2}{R_S^2} = I_S \left(\frac{R_U}{R_S} \right)^2 \tag{10}$$

Example

Both wax boxes of the Joly photometer are equally bright when the distance from the standard source is 30 cm and the distance from the unknown source is 60 cm. What is the luminous intensity of the unknown source if that of standard source equals 60 cd?

Solution:

from (10) we have: $I_U = 60\,\text{cd} \left(\frac{60\,\text{cm}}{30\,\text{cm}} \right)^2 = 240\text{ cd}$

4. Experimental procedure

4.1 Determination of the luminous intensity by the photometric method

1. Arrange two light sources on an optical bench as shown in Fig. 7. Choose one of the test bulbs as a standard. The standard bulb will be assigned a comparative luminous intensity of I_S.
2. Place the photometer between the light sources. The photometer should be at the same height as the light sources so that the lights completely illuminate the wax blocks.
3. Turn off all other lights in the room. Close any window, shades, or blinds so that only the light from the test light sources is hitting the blocks.
4. Move the photometer toward or away from the bulb taken as the standard source until both wax blocks appear equally bright.
5. Measure the distance between the photometer and each of the sources. Measure from the bulb to the interface of wax blocks.
6. Calculate the relative luminous intensity of the second bulb against your standard bulb.

4.2 Checking for the validity of the inverse square law

1. Arrange the laboratory setup as shown in Fig.8.

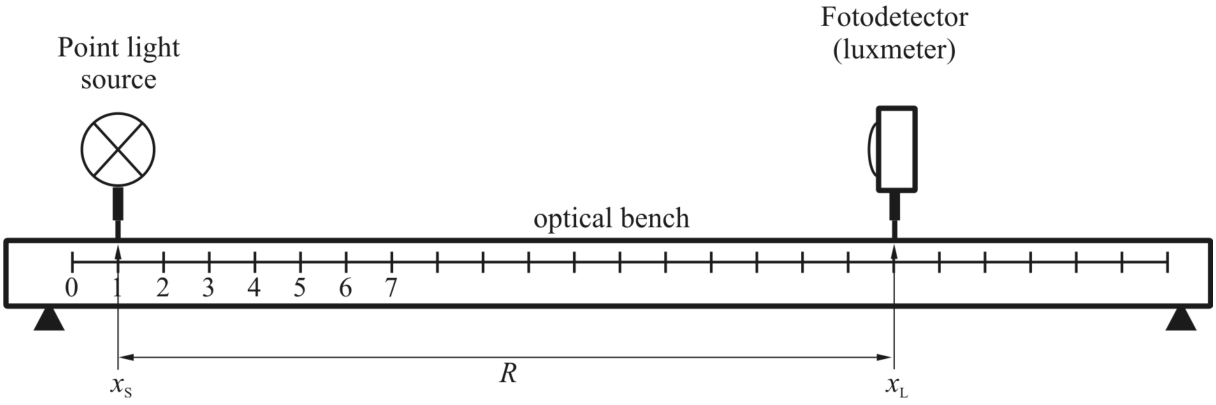


Fig. 8. A schematic diagram of the experimental setup for checking for the validity of the inverse square law.

2. Turn off all other lights in the room. Close any window, shades, or blinds so that only light from the test light sources is hitting the luxmeter.
3. Place the light source at the end of the optical bench and record its position x_S. Place the luxmeter close to the light source, record its position, x_L and the luxmeter reading E (i.e value of the illuminance).
4. Shift the lux meter to increase the distance $R = x_L - x_S$ between the light source and luxmeter, record its new position and corresponding reading E.

5. Repeat the procedure described in point 4 for different, increasing distances between the light source and luxmeter.
6. Make two graphs:
   a) $E = f(R)$ i.e. the illuminance as a function of the distance from a point light source,
   b) $E = f\left(\frac{1}{R^2}\right)$ i.e. the illuminance as a function of the inverse of the square distance from a point light source.

APPENDIX

Spectral distribution of the photopic luminous efficiency function V(λ)			
wavelength, nm	luminous efficiency V(λ)	wavelength, nm	luminous efficiency V(λ)
400	0.000	555	1.000
410	0.001	560	0.995
420	0.004	570	0.952
430	0.012	580	0.870
440	0.023	590	0.757
450	0.038	600	0.631
460	0.060	610	0.503
470	0.091	620	0.381
480	0.139	630	0.265
490	0.208	640	0.175
500	0.323	650	0.107
510	0.503	660	0.061
520	0.710	670	0.032
530	0.862	680	0.017
540	0.954	690	0.008
550	0.995	700	0.004