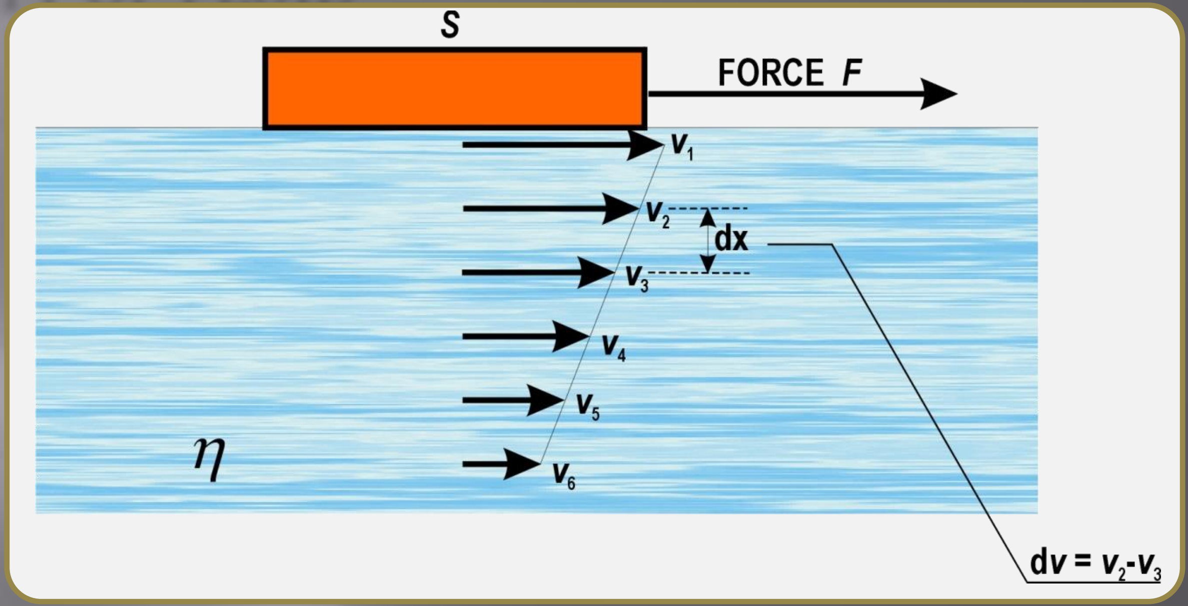
BIOPHYSICS OF CIRCULATORY SYSTEM

By PhD Marek Tuliszka
Department of Biophysics
Poznań University of Medical Sciences

VISCOSITY- A PROPERTY OF FLUID



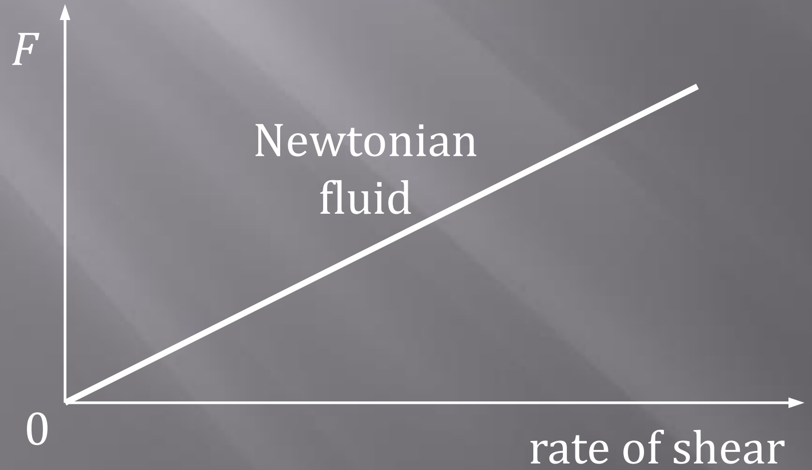
$$F = \eta \cdot S \cdot \frac{\Delta v}{\Delta x}$$

viscosity

velocity gradient
rate of shear

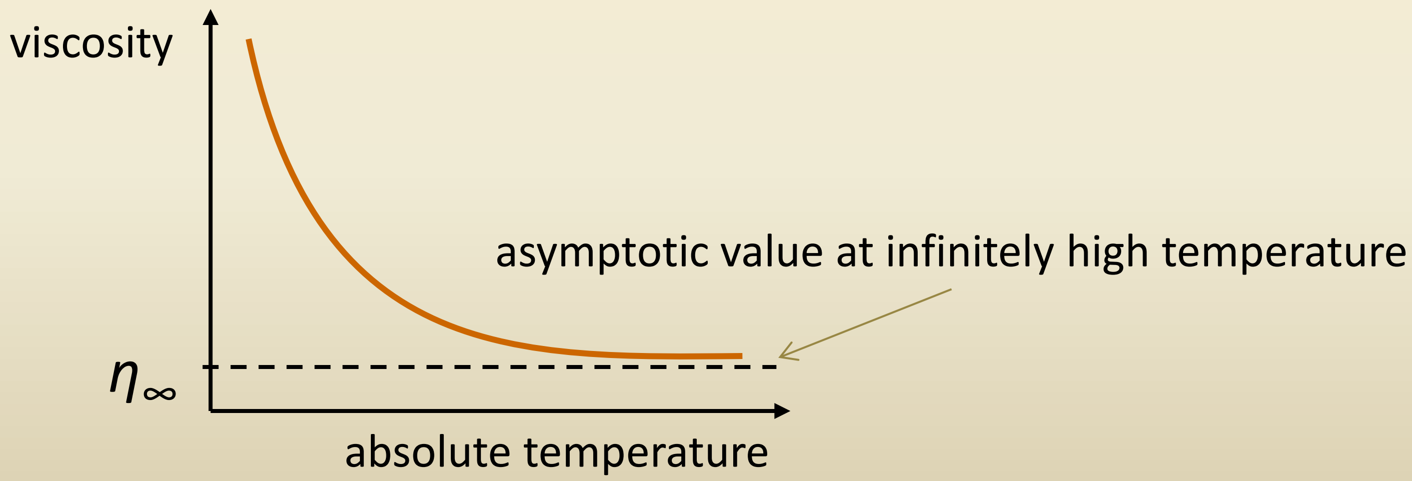
$$[\eta] = \frac{\mathrm{N \cdot s}}{\mathrm{m}^2} = \mathrm{Pa \cdot s}$$

1 P = 0.1 Pa·s
P - poise



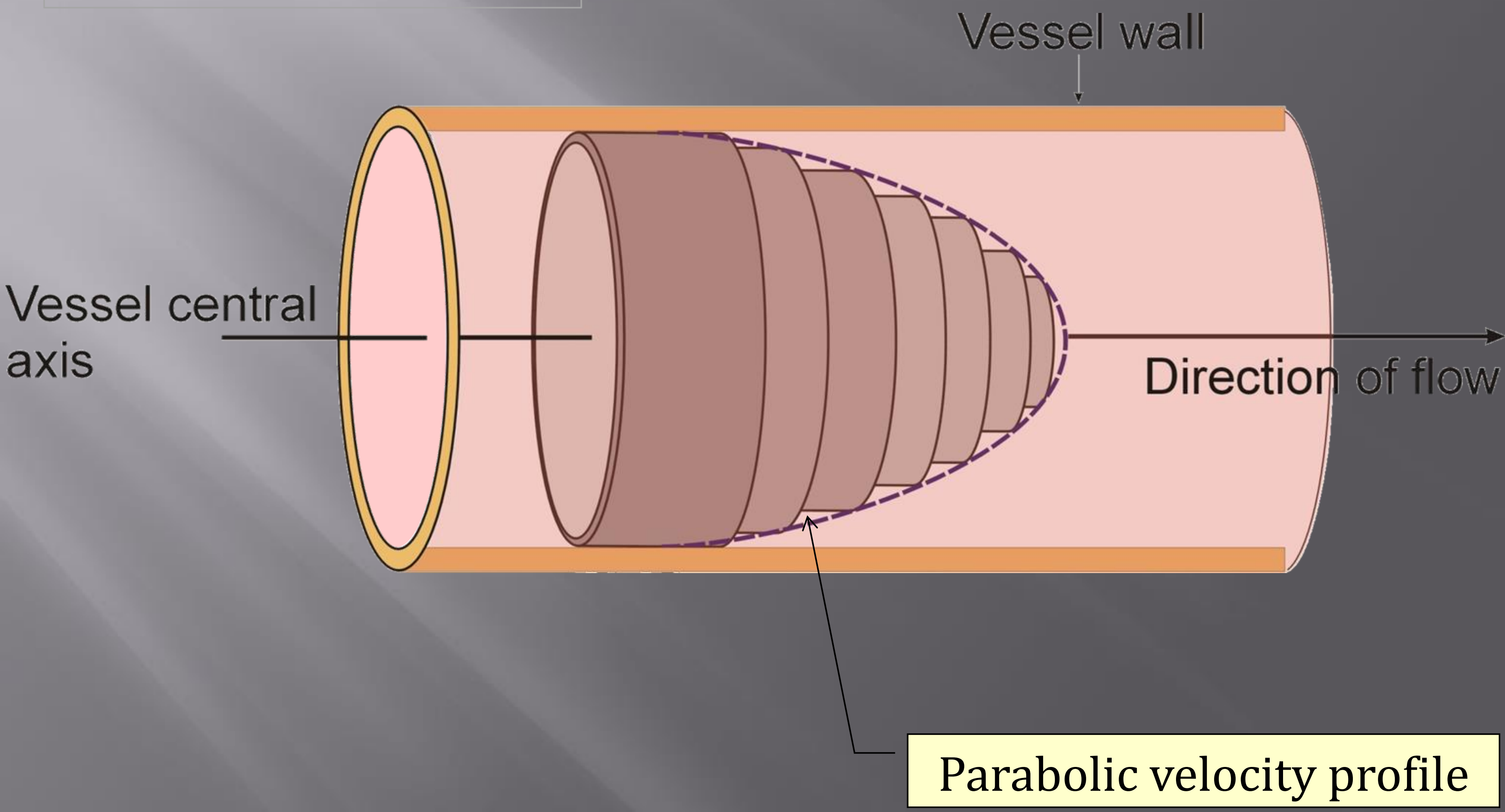
VISCOSITY DEPENDS ON TEMPERATURE

WATER	BLOOD
η_{20} = 0.0010 Pa·s (1.0 cP)	
η_{37} = 0.0007 Pa·s (0.007 P = 0.7 cP)	η_{37} = 0.0030—0.0040 Pa·s (0.04 P = 4cP)

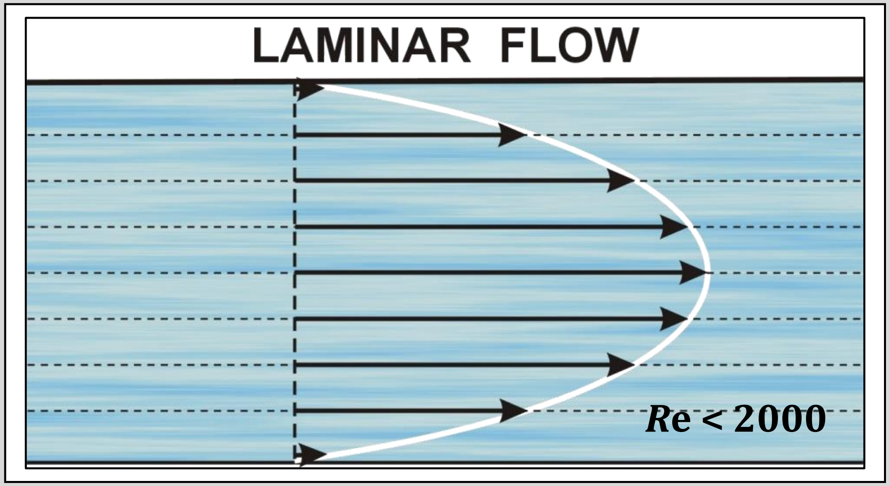


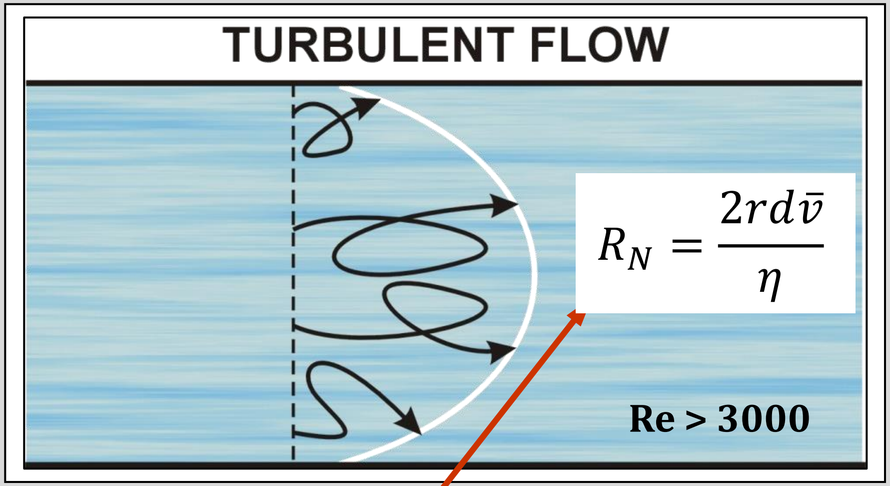
FLOW IN VESSELS

The laminar flow



THE LAMINAR FLOW VS. TURBULENT FLOW



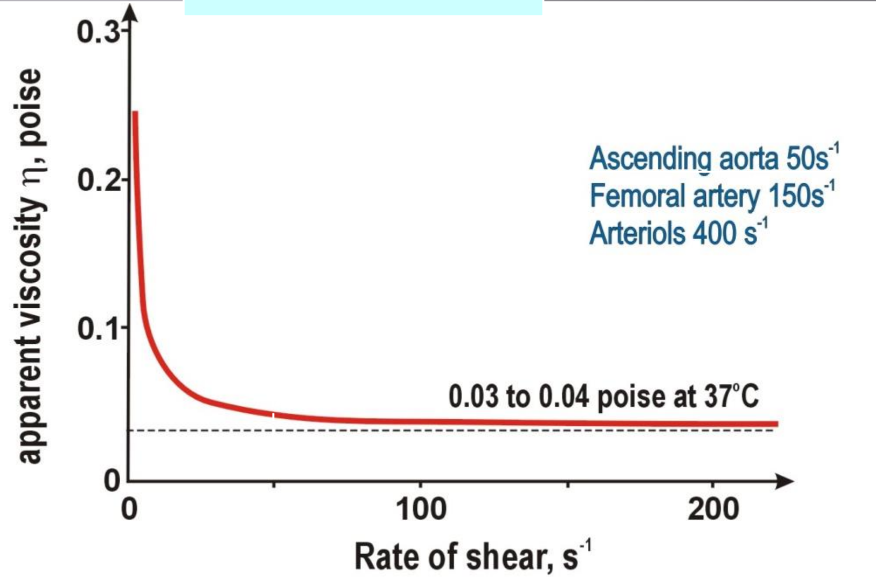


Whether turbulent or laminar flow exist in a tube under given conditions may be predicted on the basis of a dimensionless number called the **REYNOLDS NUMBER**. This number represents the ratio of inertial to viscous forces.

Viscosity of blood

Viscosity of blood depends on:

- rate of shear

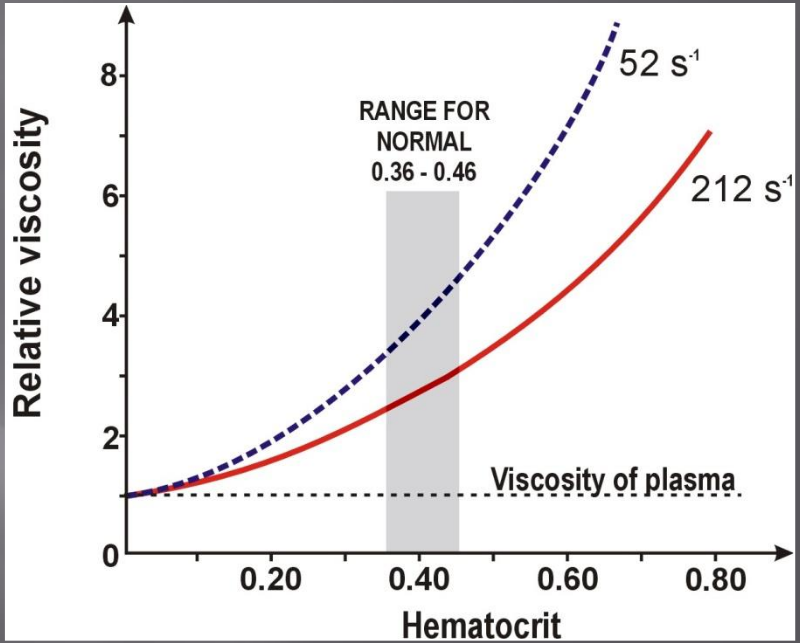


Non-Newtonian behavior of normal blood.

Viscosity (apparent viscosity) is relatively high at low rates of shear but approaches an asymptotic value above 100 sec^{-1}.

- hematocrit

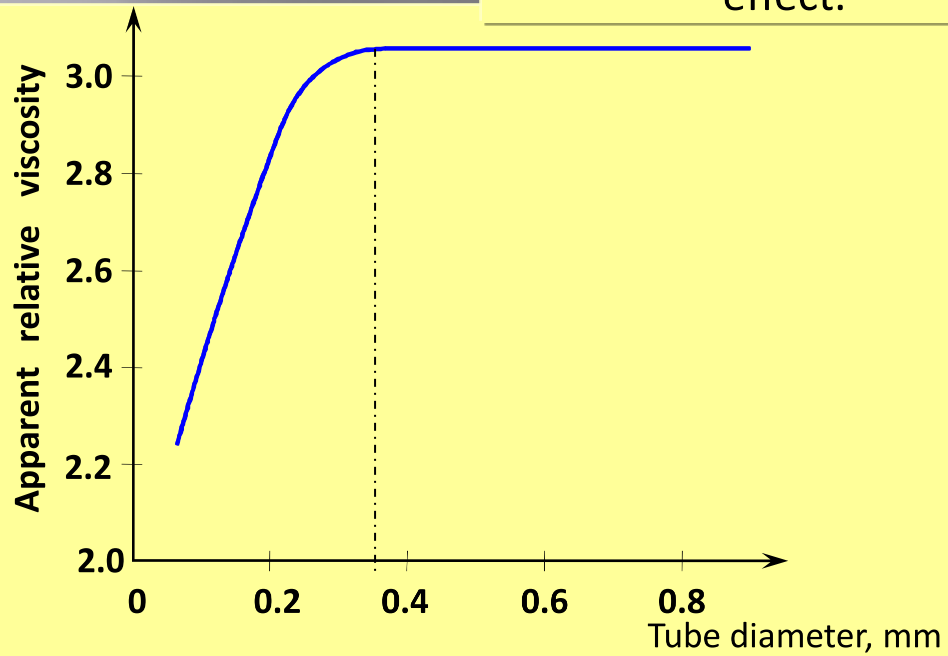
$$HCT = \frac{V_{RBC}}{V_{FULL\ BLOOD}}$$



Viscosity of blood ... depends also on:

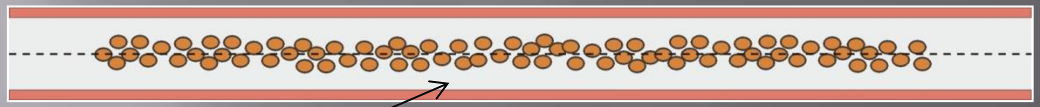
- vessel (tube) diameter

The Fåhraeus - Lindquist effect.

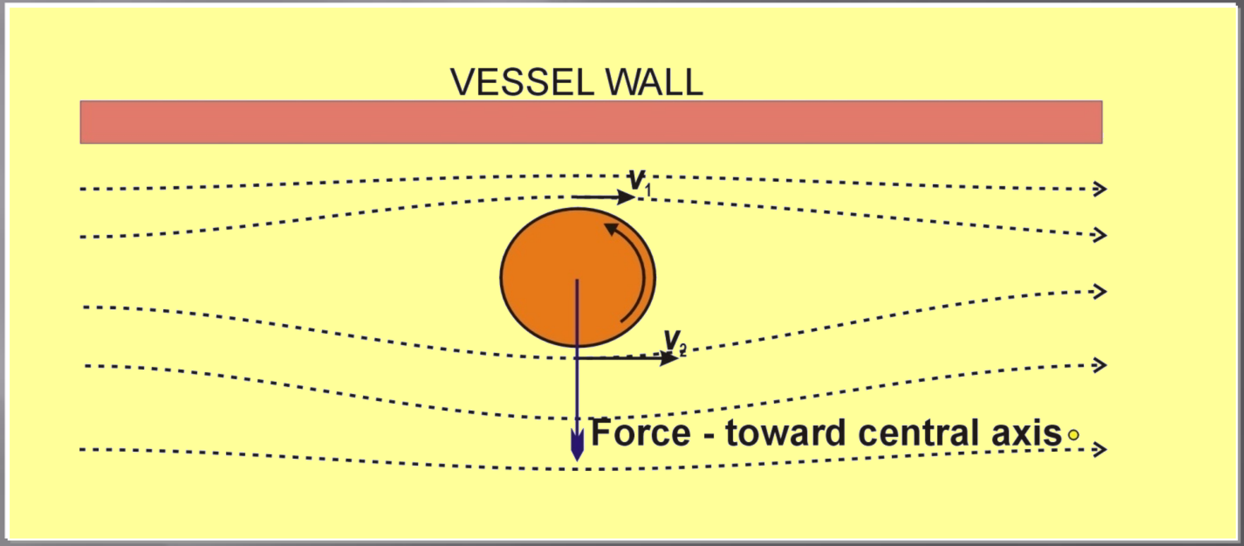


The relative, apparent viscosity of whole blood declines markedly in tubes of diameters less than approximately 0.4 mm.

AXIAL ACCUMULATION



Plasma layer of smaller viscosity (lubricant)



Consequence of the axial accumulation:

- the transit time of erythrocytes through the vascular tree is shorter than that of plasma.

LAWS OF FLUID FLOW

Laws of flow:

Requirements:

- laminar flow
- incompressible fluid
- steady flow (?)
- rigid walls (?)

- The law of continuity
- The Hagen-Poiseuille law and the *vascular resistance*
- The Bernoulli's Principle

Requirements (Bernoulli's Principle):

- non-viscous fluid (!)
- incompressible fluid

The Law of Continuity

The volume rate of flow:

$$Q = \frac{\Delta V}{\Delta t}$$

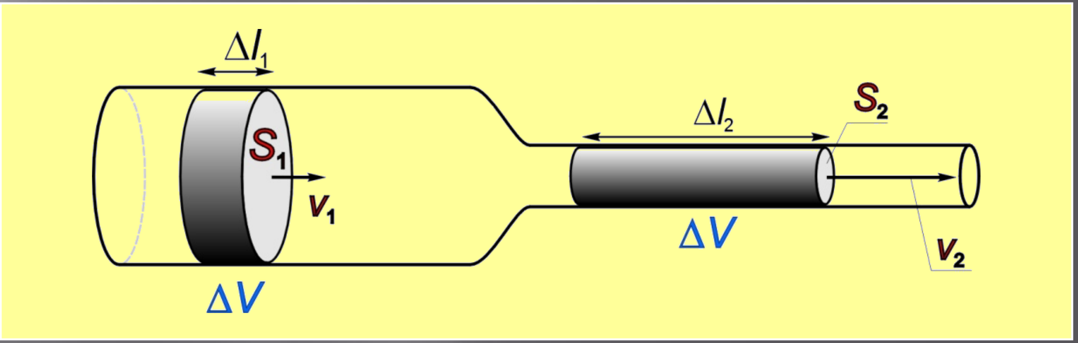
$$[Q] = \frac{\mathrm{m}^3}{\mathrm{s}} \text{ or } \frac{\text{mlitre}}{\text{s}} \text{ or } \frac{\text{litre}}{\text{minte}}$$

In the case of laminar flow of an incompressible fluid flowing through rigid tubes the volume rate of flow Q remains constant!

$$Q_1 = Q_2 = cost.$$

$$Q = \frac{\Delta V}{\Delta t} = const.$$

$$Q = \frac{\Delta V}{\Delta t} = \frac{S \cdot \Delta l}{\Delta t} = S \cdot v$$



$$S_1 \cdot v_1 = S_2 \cdot v_2 = const.$$

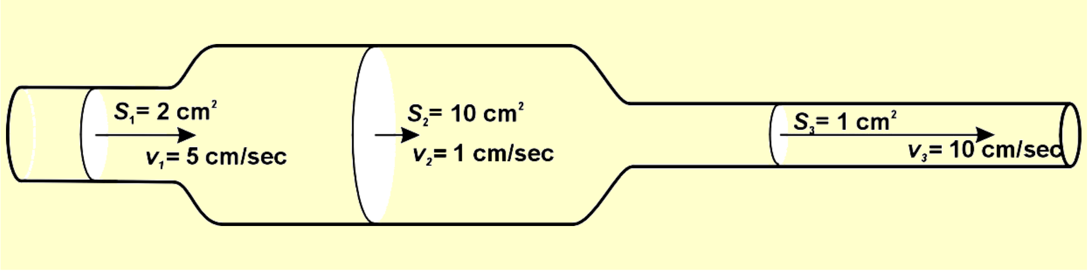
$$\frac{v_1}{v_2} = \frac{S_2}{S_1}$$

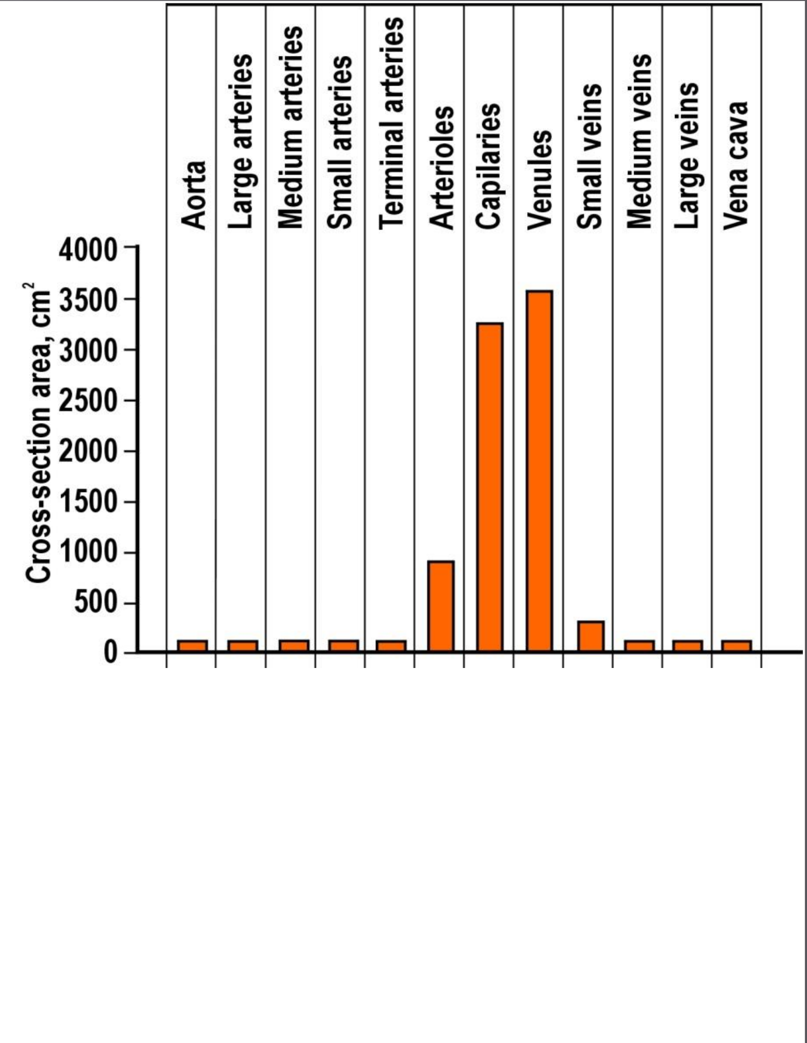
For any given flow the ratio of the velocity past one cross section relative to that past a second section depends only on the inverse ratio of the respective areas.

The Law of Continuity - conclusions

If the cross-sectional area of a tube increases the velocity of flowing liquid decreases and inversely.

$$S_1 \cdot v_1 = S_2 \cdot v_2 = S_3 \cdot v_3$$

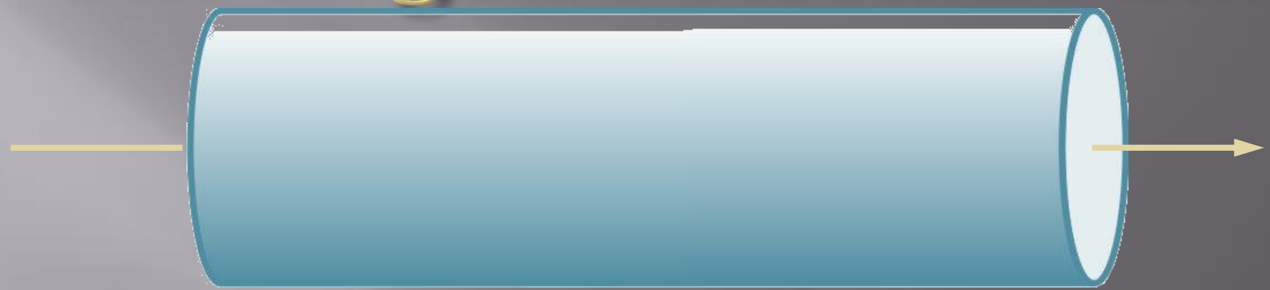




For the same volume of fluid per second passing from section area S_1 to section area S_2, which is five times greater, the velocity of flow diminishes to one fifth of its previous value.

Changes in blood velocity in the system of circulation:

The Hagen-Poiseuille Law



What factors affect the volume rate of flow Q through the tube above?

- the pressure difference between the ends of the tube Δp
- the viscosity of the fluid η
- the tube radius r
- the tube length l

$$Q = \frac{\pi r^4}{8\eta l} \cdot \Delta p$$

The analogy:

The Δp factor can substitute for the difference in the mean arterial systolic pressure *MAP* and the mean right diastolic ventricular pressure *MVP*

$$CO = \frac{\pi r^4}{8\eta l} \cdot (MAP - MVP)$$

Q is the analogue of the cardiac output *CO*

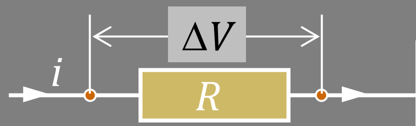
The Hagen-Poiseuille Law and the vascular resistance

$$Q = \frac{\pi r^4}{8\eta l} \cdot \Delta p$$

$Q \rightarrow i \qquad \Delta p \rightarrow \Delta V$

By analogy with Ohm's law

$$i = \frac{\Delta V}{R}$$



$$i = \frac{\Delta q}{\Delta t}$$

$$i = \frac{1}{R} \cdot \Delta V \quad \Longleftrightarrow \quad Q = \frac{\pi r^4}{8\eta l} \cdot \Delta p$$

The hydraulic or vascular resistance:

$$R_V = \frac{8\eta l}{\pi r^4}$$

$$R_V = \frac{\Delta p}{Q}$$

Unit of R_V: $\dfrac{\text{Pa} \cdot \text{s}}{\text{m}^3}$

Unit of R_V: $\dfrac{1\ \text{mmHg}}{\frac{1\ \text{mliter}}{\text{min}}}$

$$Q = \frac{1}{R_V} \cdot \Delta p$$

Application in physiology

The fundamental equation of cardiovascular physiology

$$Q = \frac{1}{R_V} \cdot \Delta p$$

$$CO = \frac{MAP}{SVR}$$

SVR – **S**ysytemic **V**ascular **R**esistance
CO – cardiac output
MAP – mean arterial pressure

Unit of R_V: $\frac{1\ \text{mmHg}}{\frac{1\ \text{mliter}}{\text{min}}}$

Vascular resistance – examples:

$$\boldsymbol{Q = \frac{1}{R_V} \cdot \Delta p}$$

$$FLOW = \frac{DIFFERENCE\ IN\ PRESSURE}{RESISTANCE\ TO\ FLOW}$$

$$RESISTANCE\ TO\ FLOW = \frac{DIFFERENCE\ IN\ PRESSURE}{FLOW}$$

Example: Cardiac output (CO) through the circulatory system, when a person is at rest, is 100 ml/s (6000 ml/min.)

and

the pressure difference from the systemic arteries to the systemic veins is approximately about 100 mmHg. Therefore the resistance of the entire system, *SVR* (the systemic vascular resistance) is approximately equal to:

$$SVR = \frac{100\ \mathrm{mmHg}}{6000\,\frac{\mathrm{ml}}{\mathrm{min}}} = 0.017\ \frac{mmHg}{\frac{\mathrm{ml}}{\mathrm{min}}} = 0.017\,\frac{\mathrm{mmHg}\cdot\mathrm{min}}{\mathrm{ml}}$$

Example: The brain has flow 750 ml per minute, the pressure difference is 100 mmHg. Determine the cerebrovascular resistance (CVB):

$$R_{\mathrm{CVB}} = \frac{100\ \mathrm{mmHg}}{750\ \frac{\mathrm{ml}}{\mathrm{minute}}} = 0.13\ \frac{\mathrm{mmHg{\cdot}min}}{\mathrm{ml}}$$

Example: The lungs have flow 100 ml/s, mean pulmonary arterial pressure is 16 mmHg and mean left atrial pressure is 2 mmHg, thus:

$$R_{\mathrm{lungs}} = \frac{16\ mmHg - 2\ \mathrm{mmHg}}{6000\ \frac{\mathrm{ml}}{\mathrm{minute}}} = 0.0023\ \frac{\mathrm{mmHg{\cdot}min}}{\mathrm{ml}}$$

NOTE:
If the difference of pressure at the end terminals of a system is smaller, whereas the rate of flow is unchanged the vascular resistance shown by the system is : …………………...

$$\boldsymbol{Q} = \frac{\mathbf{1}}{\boldsymbol{R}_{\mathbf{V}}} \cdot \Delta\boldsymbol{p}$$

$$R_{\mathrm{V}} = \frac{\Delta\boldsymbol{p}}{Q}$$

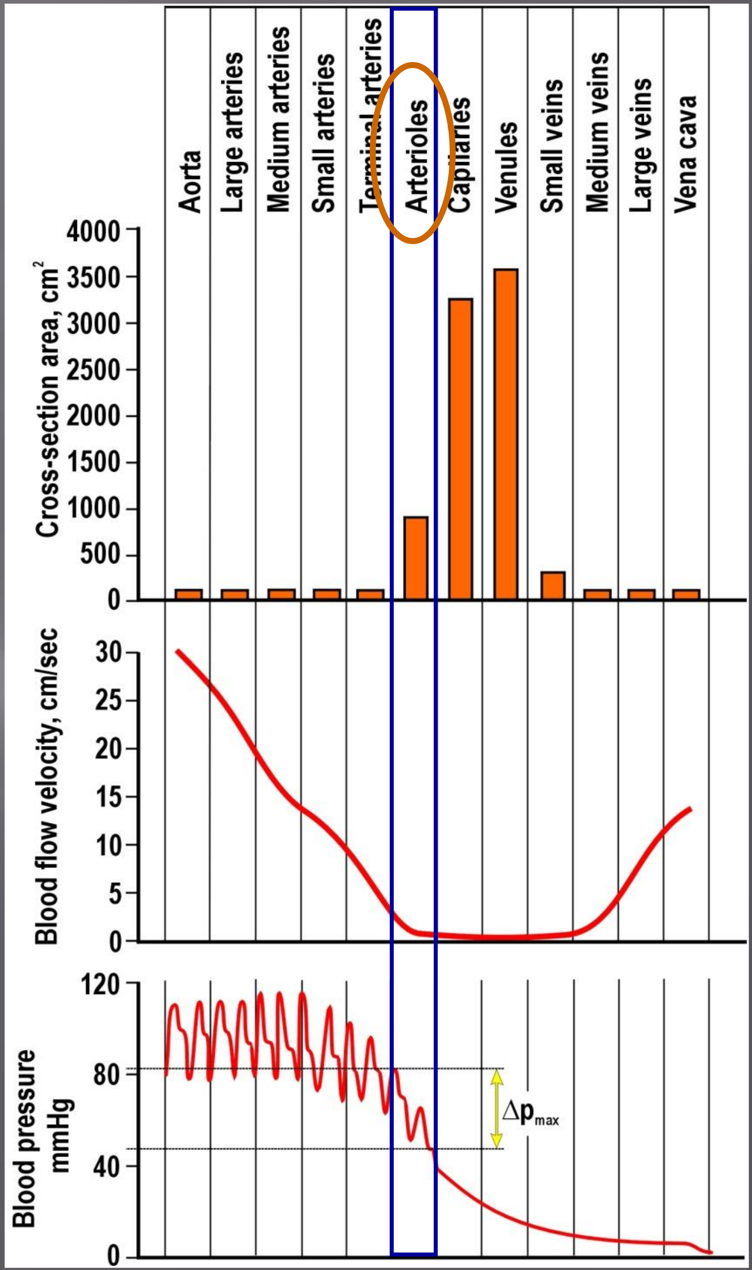
Vascular resistance – conclusions

$$Resistance = \frac{\Delta p}{Flow}$$

Since total flow is the same through the various series components of the circulatory system, the greatest resistance to flow resides in the arterioles (the greatest drop in pressure).

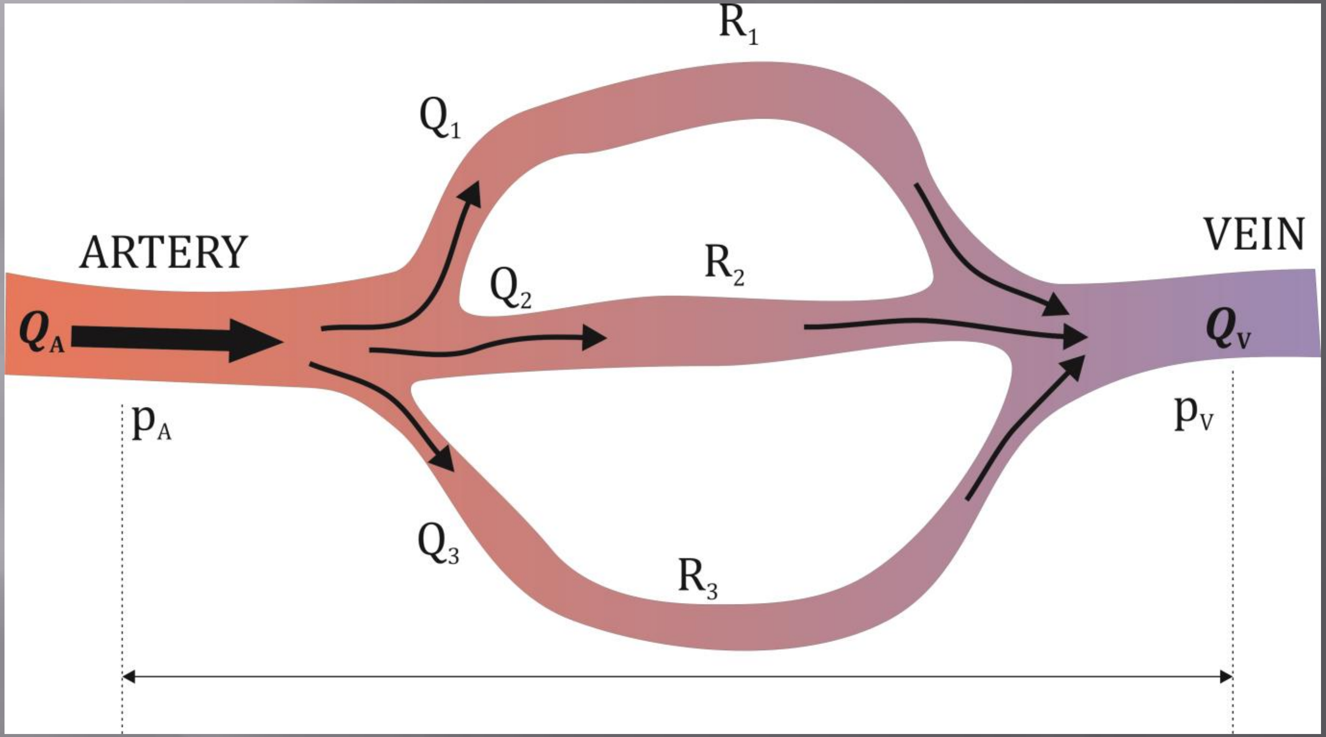
$$R_V = \frac{8\eta l}{\pi r^4}$$

$$geometrical\ factor = \frac{l}{r^4}$$



Arrangement of blood vessels:

- parallel circuit

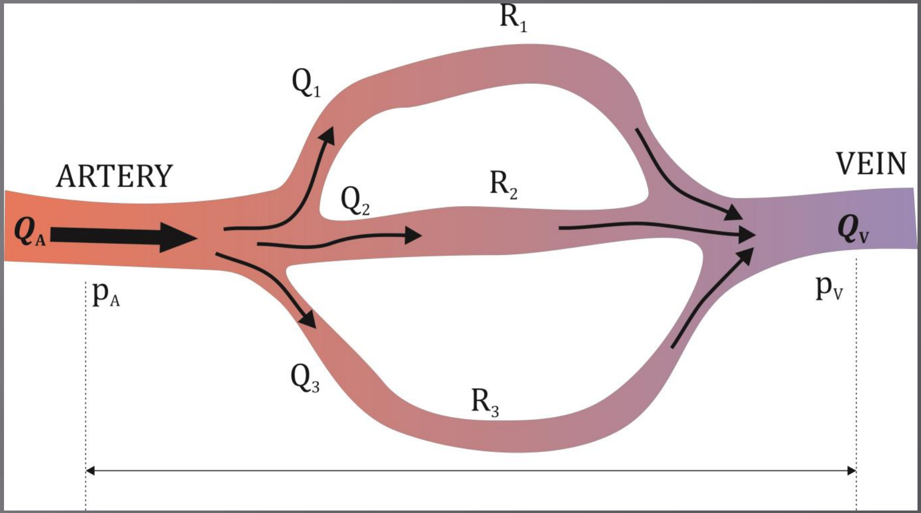


$$Q_A = Q_V = Q_T$$

$$Q_T = Q_1 + Q_2 + Q_3$$

$$Q_\mathrm{T} = Q_1 + Q_2 + Q_3$$

$$Q = \frac{\Delta p}{R}$$



$$\Delta p = p_\mathrm{A} - p_\mathrm{V}$$

$$\frac{p_\mathrm{A} - p_\mathrm{V}}{R_{eq.}} = \frac{p_\mathrm{A} - p_\mathrm{V}}{R_1} + \frac{p_\mathrm{A} - p_\mathrm{V}}{R_2} + \frac{p_\mathrm{A} - p_\mathrm{V}}{R_3}$$

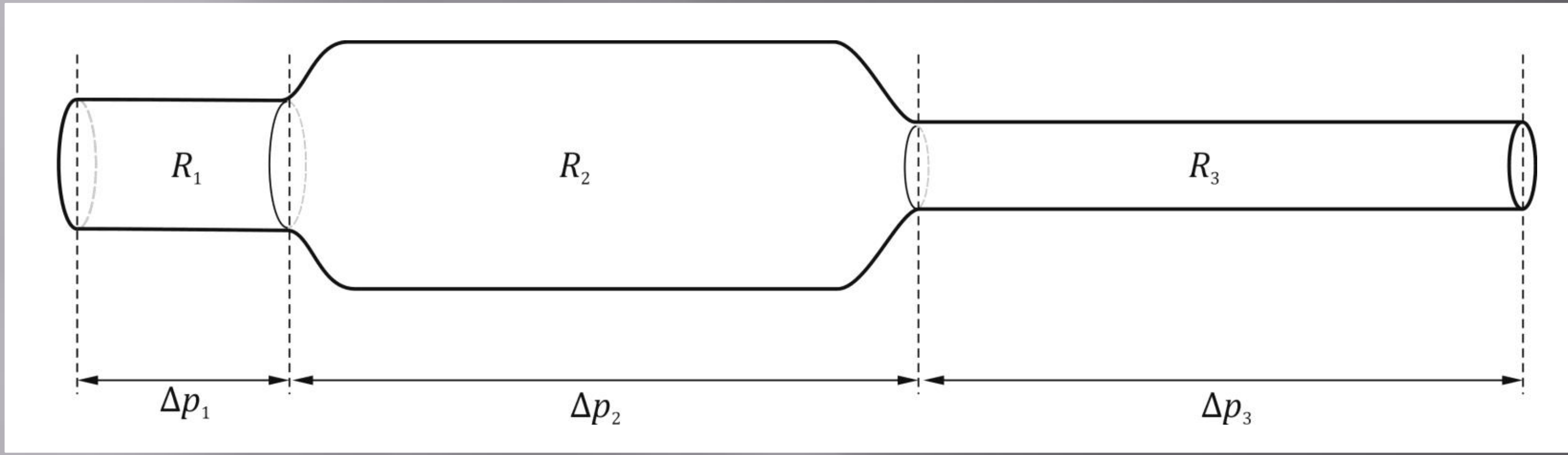
$$\frac{\mathbf{1}}{\boldsymbol{R_{eq.}}} = \frac{\mathbf{1}}{\boldsymbol{R_1}} + \frac{\mathbf{1}}{\boldsymbol{R_2}} + \frac{\mathbf{1}}{\boldsymbol{R_3}}$$

$R_1 = 3$ u.
$R_2 = 2$ u.
$R_3 = 4$ u.

$$\frac{1}{\boldsymbol{R_{eq.}}} = \frac{1}{3\text{ u.}} + \frac{1}{2\text{ u.}} + \frac{1}{4\text{ u.}} = \frac{13}{12\text{ u.}}$$

$$\boldsymbol{R_{eq.}} = \frac{12\text{ u.}}{13} = 0.92\text{ u.}$$

➢ series circuit



$$\Delta p = \Delta p_1 + \Delta p_2 + \Delta p_3$$

The pressure drops along the streamline due to frictional flow of *viscous fluid*.

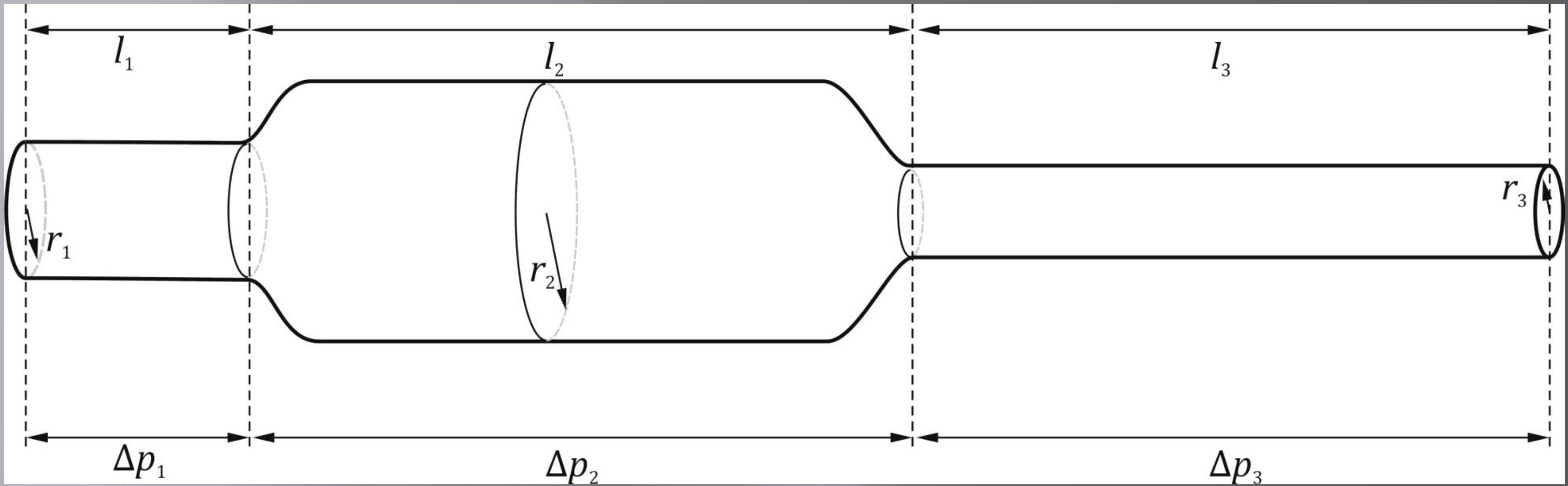
$$Q = \frac{\Delta p}{\boldsymbol{R_{eq.}}}$$

$$\Delta p = Q \cdot \boldsymbol{R_{eq.}}$$

$$Q \cdot \boldsymbol{R_{eq.}} = Q \cdot R_1 + Q \cdot R_2 + Q \cdot R_3$$

$$\boldsymbol{R_{eq.} = R_1 + R_2 + R_3}$$

EXAMPLE (series circuit)



$l_1 = l \quad l_2 = 2l \quad l_3 = 2l$

$r_1 = r \quad r_2 = 2r \quad r_3 = 0.5r$

Task: Compare values of the vascular resistance of the segments shown in the drawing.

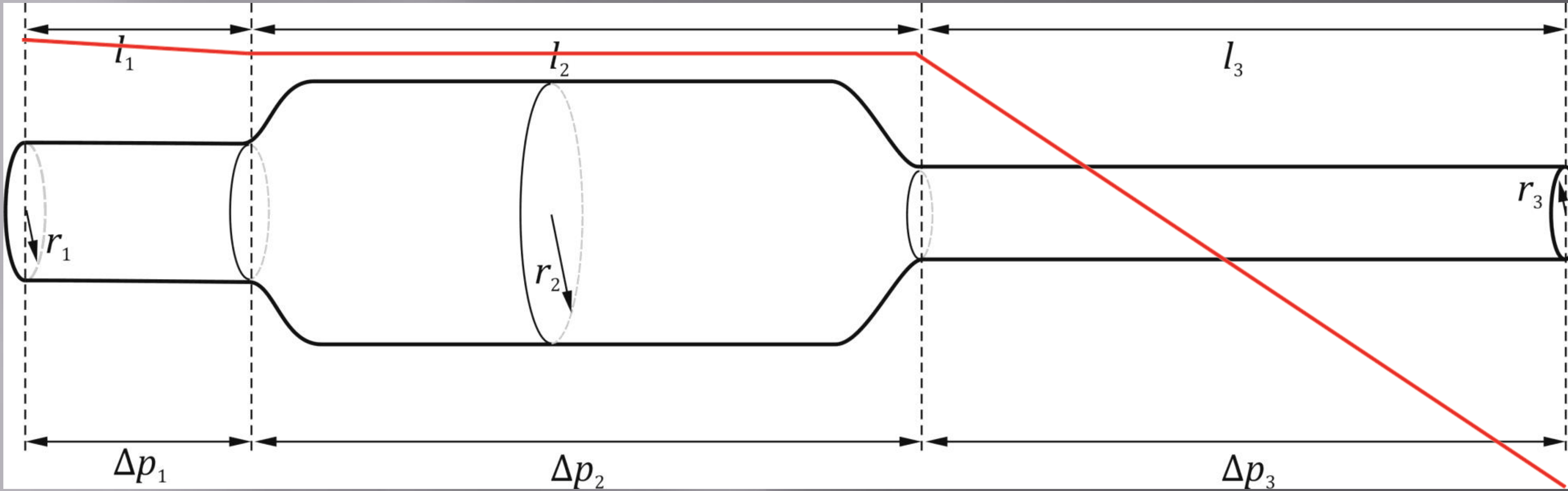
$$R_{V1} = \frac{8\eta l}{\pi r^4}$$

$$R_{V2} = \frac{8\eta 2l}{\pi (2r)^4} = \frac{2}{16} \cdot \frac{8\eta l}{\pi r^4} = \frac{1}{8} R_{V1}$$

$$R_{V3} = \frac{8\eta 2l}{\pi (0.5r)^4} = \frac{2}{\frac{1}{16}} \cdot \frac{8\eta l}{\pi r^4} = 32 R_{V1}$$

Task:
Assuming R_{V1} equals R, determine total vascular resistance of the system in terms of R.

$$\boldsymbol{R_{Ts.} = R_1 + R_2 + R_3 = R + 0,125R + 32R = 33.125R}$$



$l_1 = l \quad l_2 = 2l \quad l_3 = 2l$

$r_1 = r \quad r_2 = 2r \quad r_3 = 0.5r$

Compare drops in pressure along each segment of the system

$\Delta p_1 = Q \cdot R_{V1}$

$$geometrical\ factor_1 = \frac{l}{r^4}$$

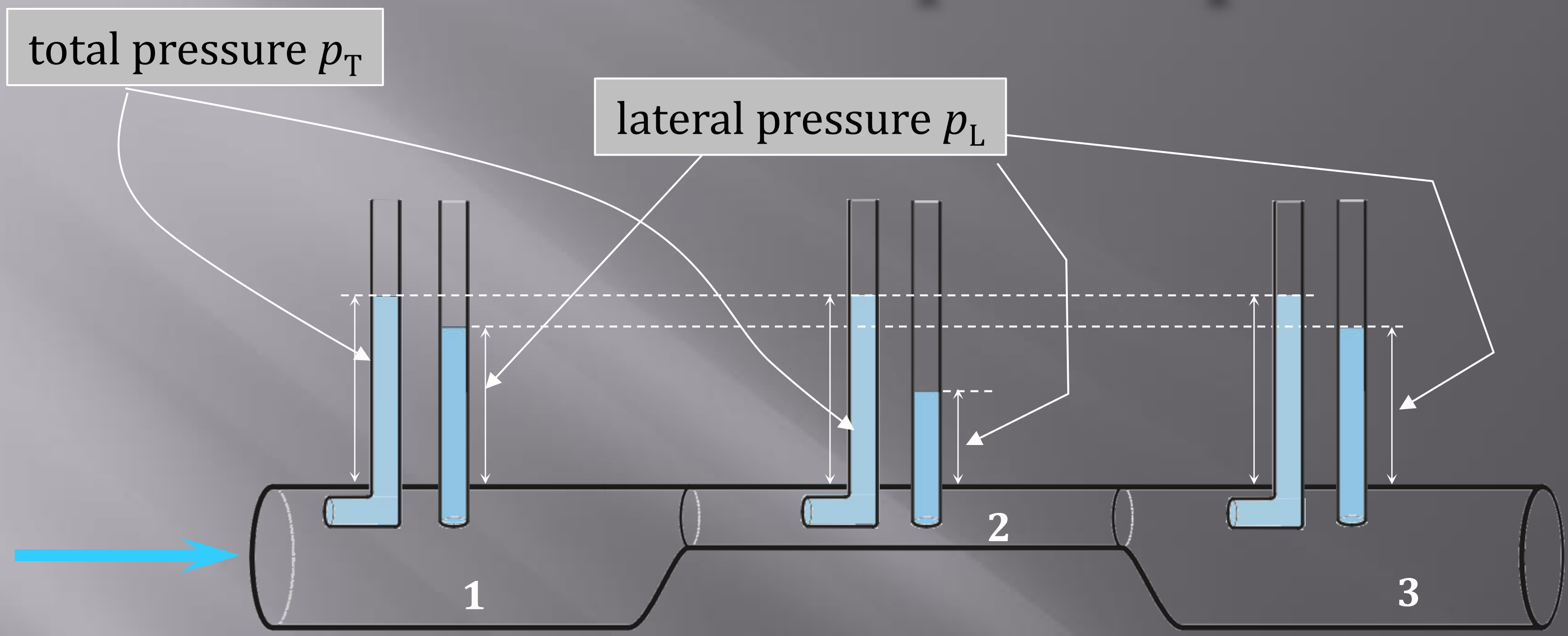
$\Delta p_2 = \frac{1}{8} Q \cdot R_{V1}$

$$geometrical\ factor_2 = \frac{2l}{(2r)^4} = \frac{1}{8}\frac{l}{r^4} = 0{,}125\frac{l}{r^4}$$

$\Delta p_3 = 32Q \cdot R_{V1}$

$$geometrical\ factor_3 = \frac{2l}{(0.5r)^4} = 32\frac{l}{r^4}$$

Bernoulli's principle



For non-viscous fluid (!):

TOTAL PRESSURE = LATERAL (STATIC) PRESSURE + DYNAMIC PRESSURE = CONSTANT

$p_{T1} = p_{T2} = p_{T3} = \text{const.}$

$p_{L1} + p_{D1} = p_{L2} + p_{D2} = p_{L3} + p_{D3} = \text{const.}$

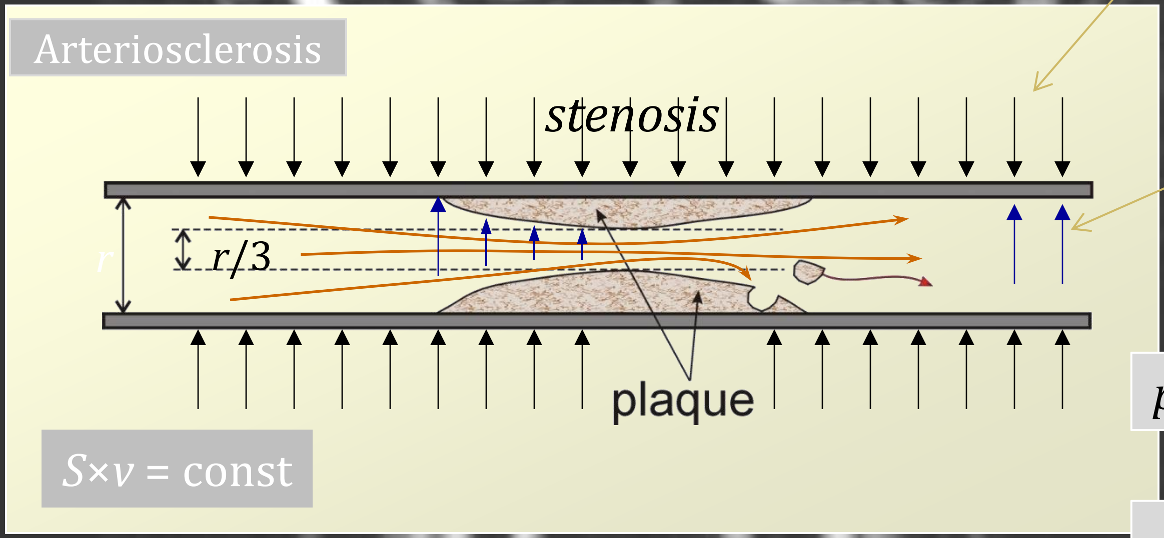
$$p_D = \frac{1}{2} d \cdot v^2$$

d – density of fluid
v – velocity of fluid

Consequences of Bernoulli's Principle

external „tissue" pressure

Lateral (static) blood pressure



$p_{\text{lateral}} + p_{\text{dynamic}} = const.$

$$p_D = \frac{1}{2} d v^2 \quad (S = \pi r^2) \rightarrow v \times 9$$

corresponding decrease of the lateral pressure!

$$R_V = \frac{8\eta l}{\pi r^4} \longrightarrow \left(\frac{1}{\frac{1}{3}}\right)^4 = 81$$

Cervical arteries

(X-rays with contrast media)

BMRI © J.Hornak

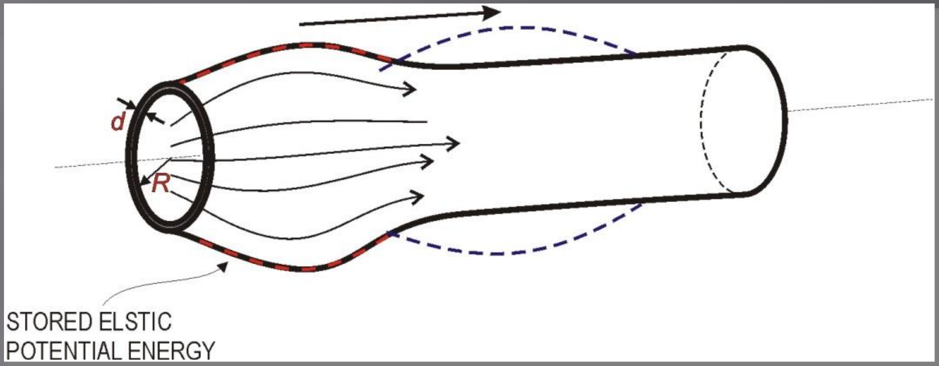
Distensibility of blood vessels - the pulse wave

Pulse wave in distensible vessels

$$v_p = \sqrt{\frac{K}{\rho}}$$

K – the bulk modulus of elasticity
ρ - density of flowing fluid,

The *bulk modulus of elasticity* ***K*** is defined as the ratio of change in pressure $\boldsymbol{\Delta p}$ and the relative change in a vessel volume ($\mathbf{\Delta V/V_0}$) resulting from this change:



E - Young's modulus of wall material
d - wall thickness
R - vessel radius

$$K = \frac{\Delta p}{\frac{\Delta V}{V_0}} = \frac{Ed}{2R}$$

The velocity V_P of a puls wave:
Moens- Korteweg equation:

$$v_p = \sqrt{\frac{K}{d}} = F\sqrt{\frac{Ed}{2R\rho}}$$

F - correction factor (due to viscosity of fluid and presence of surrounding tissues)

Typical values of the pulse wave velocity are: 5-8 m/s.

↑

The Young modulus for aorta more than doubles between the ages of 20 and 60 years.

↓

Typical value of the velocity of blood flow in aorta is less than 0.5 m/s.